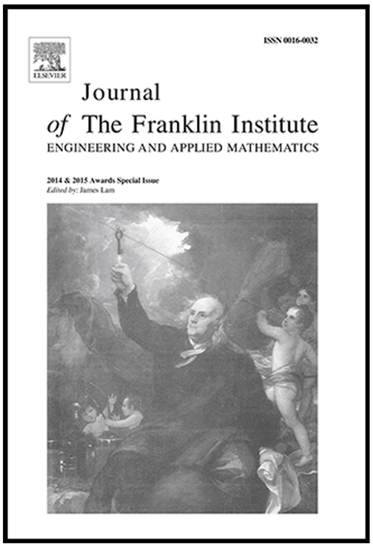

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Event-driven Finite-Time Control for Continuous-time Networked Switched Systems under Cyber Attacks

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Abstract

This paper addresses the issue of event-driven state-feedback controller design for networked switched systems in the finite-time sense. At the plant design level, the synchronism between switched systems and the corresponding controllers is guaranteed by introducing a mode-decision unit at the actuator side. Through intensive time interval analysis approach, the switched system, the event-triggered scheme, transmission delays and cyber attacks are unified into a new closed control system. Sufficient conditions are developed to ensure the networked switched system finite-time bounded by constructing a proper Lyapunov function. Then, the design methods for switched sub-controllers, optimal event-triggered parameters and restrictive average dwell-time switching law are presented. It is worth pointing out that the main results not only contribute to optimize limited network resources but also improve the system security level, which is validated through an illustrative example of a boost converter circuit.

Keywords: Event-triggered mechanism; Switched system; Finite-time stability; Cyber attack; Network-induced delay

1. Introduction

In recent years, networked control systems (NCSs) have experienced an increasing attention owing to the advantages of low cost, high flexibility, convenient installation and maintenance [1–4]. Nevertheless, the characteristic of network may bring some intractable problems, i.e., network-induced delay, data loss and disorder, network congestion, and so on. To optimize the network resources, event-triggered mechanisms (ETMs) in various forms have been developed [1, 5, 6], which take the veritable needs of control systems into consideration and provide an effective solution to exclude the Zeno behaviour [7]. The authors in [8] presented a network-based model for offshore structures and investigated the relevant event-driven H_∞ reliable control problem. [9] and [10] were concerned with the event-driven H_∞ filtering for neural networks and Markov jump systems. [11] and [12] studied the decentralized filtering or control problems for NCSs with event-based strategies. In [13], the probabilistic-constrained filtering problem was investigated under an improved event-triggered scheme. [14] addressed the event-based consensus tracking problem for higher order stochastic nonlinear multiagent systems. Although the effectiveness of ETMs is verified adequately, how to design the specific event-triggered schemes for various application systems is a problem deserving of further investigation.

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As a kind of hybrid systems, the switched system, has drawn sufficient attention of researchers due to their favourable potential in practical applications, such as robot control systems [15], electronic circuits [16], and aircraft control systems [17]. Recently, it is a hot topic to combine switched systems with other control systems, for instances, multiagent systems [18, 19] and NCSs [20–25], especially with event-driven strategies. It is not just a simple superposition of the two systems while the combination brings more interesting and challenging problems due to its hybrid nature. The focus is layed not only on the establishment of physical plants but also the design of controllers. The event-triggered control of switched linear systems was early studied in [26]. In the framework, no more than one switching was permitted within a defined trigger interval, which resulted in too much conservativeness. [21] was concerned with the event-triggered finite-time stability of networked switched systems, in which the mode and state information of switched systems were sent into the controller through separate channels. In [22], the stabilisation of switched linear systems was analysed by a fixed-value event-triggered mechanism. Nevertheless, the network-induced delay is not taken into consideration in [20–22], which is of practical significance. Relatively, [23] , [24], and [25] took the transmission delay and asynchronous switching phenomenon into consideration. The mode and state information of switched systems are sent into the controllers through networks simultaneously, which inversely makes the modeling process rather complex. It is worth pointing out that the research work for networked switched systems is still far from being fully investigated, which is the main motivation of this paper.

In terms of stability control theory, primary research work in the literature is focused on conventional Lyapunov asymptotic stability (LAS). Nevertheless, in some specific applications, such as military systems, aero systems, etc, LAS is not satisfactory while short-time stability should be guaranteed in the first place. That is, the system is Lyapunov stable but disabled due to transient uncertainties. In this view, a potential analytical method of finite-time boundedness (FTB) was developed in [27], and more and more comprehensive attentions have been taken to FTB nowadays [28–32]. In regards to networked switched systems concerned in this paper, some preliminary work has been conducted in [20–23]. Nevertheless, it is still largely open for the design of event-triggered controllers for networked switched systems in the finite-time sense.

Compared with conventional systems, NCSs are vulnerable to cyber attacks due to the specific usage of communication networks [33]. To ensure the transmission efficiency, the sampled data is usually transmitted over network without security mechanisms, which gives possiable access to attackers. Much emphysis has been placed on the security control for NCSs [34–38]. The authors in [36] investigated the event-triggered H_∞ control for multiarea power systems under hybrid cyber attacks. [37] was concerned with the resilient filter design for cyber-physical systems under deception attacks. [38] pointed out that smart grid with hybird communication networks was more vulnerable to hybrid cyber attacks. Generally, the deception attack can easily degrade the system performance and escape from the existing attack detection mechanism due to its distinctive nature. Hence, the deception attack brings a huge challenge for the design of controllers. However, when it comes to the issue of networked switched systems, few results in the literature are available till now, which also motivates us in this paper.

This paper devotes to the event-driven controller design of networked switched systems in the finite-time sense. The main contributions of this paper are highlighted as follows: (1) A novel event-triggered communication scheme is constructed with the mode and state information of switched systems sent separately in the physical plant. It provides a solution to exclude the asynchronous switching phenomenon; (2) Through intensive time interval analysis approach, the switched system, the event-triggered scheme, cyber attacks and transmission delays are unified into a new closed control system; (3) Sufficient conditions for FTB are presented with the controllers and the weight of ETM developed by Lyapunov theory, and the effectiveness is verified through an example of a boost converter circuit.

The remainder of the paper is organized as follows. Section 2 introduces the concerned system and presents the co-design of the controller and ETM. In Section 3, sufficient conditions for finite-time boundedness are derived, and then the state-feedback controller design is performed. Section 4 demonstrates the effectiveness of theoretical results by an illustrative example.

Notations: $\mathbb{R}^n$ refers to the n dimensional Euclidean space while $\mathbb{R}^{n\times m}$ denotes a $m \times n$ real matrices. $diag\{A, B\}$ represents the block-diagnal matrix. $\| \cdot \|$ stands for the Euclidean vector norm. For a symmetric matrix W, $\lambda_{min}(W)$ and $\lambda_{max}(W)$ refer to the minimum and maximum eigenvalue of matrix W. $\ell_2[0,\infty)$ means the space of square-integrable vector functions over $[0,\infty)$. So, $\forall\omega(t) \in \ell_2[0,\infty)$, $\omega(t)$ can be defined by $\| \omega(t) \|_2 = \sqrt{\int_0^\infty \omega^T(t)\omega(t)dt}$.

2. Problem Formulation

In this part, we first illustrate the control framework for a networked swithed system. Taking the characteristics of the switched system into consideration, an appropriate event-triggered scheme is designed under network-induced delays and cyber attacks. Then, an unified model for the switched system is presented.

2.1. Physical Plant

In this paper, the following switched linear system is considered:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ represents the state-feedback control input while $\omega(t) \in \ell_2[0,\infty)$ denotes the disturbance input vector. $A_{\sigma(t)}$, $B_{\sigma(t)}$, $E_{\sigma(t)}$ are known real matrices with appropriate dimensions. $\sigma(t) : [0,\infty) \mapsto £ = \{1, 2, \cdots, L\}$ is a sectional-continuous function representing the switching signal, which can be further defined as $\sigma(t) = \{ (l_0, \sigma(l_0)), (l_1, \sigma(l_1)), \cdots, (l_q, \sigma(l_q)), \cdots\}$ with $l_0 = 0$. $\{l_q\}$ are switching instants. That is, when $t \in [l_q, l_{q+1})$, the subsystem $\sigma(l_q)$ is active.

As shown in Fig. 1, the sampled state information is sent to the controller over a wireless network, which is vulnerable to cyber attacks. To optimize communication resources, an event-triggered mechanism with proper triggering conditions needs to be designed. Once an event occurs, the sampled data is sent to the controller. The updated control singnal is connected to the actuator and holden by the zero-order-hold (ZOH) till next data coming. To simplify and clarify the exposition below, the following assumptions are made.

Assumption 2.1. *For a given interval $[0, T_f]$, the disturbance $\omega(t)$ is restricted by $\int_{t_0}^{T_f} \omega(t)^T\omega(t)dt \leqslant H_M$.*

Assumption 2.2. *As shown in Fig. 1, the mode-detection information is sent to the actuator by wire transmission and the frequency of mode detection is much higher than data sampling. Considering the wide usage of digital controllers, the control information of all subsystems is produced*

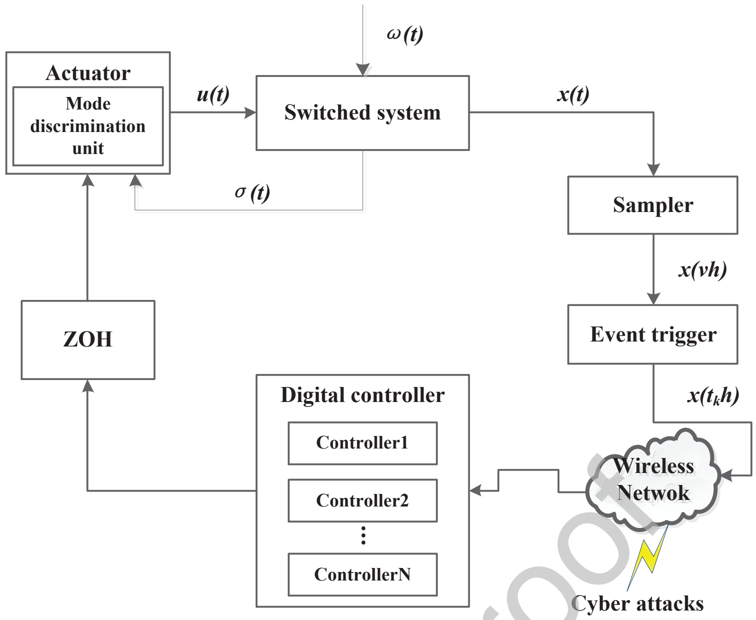

Figure 1: System plant

and sent into the actuator. In other words, a mode-decision unit is introduced before the actuator to distinguish the executive state-feedback controller gain. Based on this assumption, the asynchronous phenomenon between the subsystem and its corresponding controller is out of scope of this paper.

2.2. Cyber attacks

A type of deception attacks, namely false data injection attack, is considered in this framework. Then, the state-feedback controller under the false data injection attack can be expressed as:

$$u(t) = K_{\sigma(t)}(x(t) + \zeta(t)) \tag{2}$$

where $K_{\sigma(t)}$ is the corresponding feedback controller gain of each subsystem to be designed.

Assumption 2.3. *Suppose that the introduced false data injection attack $\zeta(t)$ satisfies the following condition:*

$$\| \zeta(t) \|_2 \leqslant \| Gx(t) \|_2 \tag{3}$$

where G is a given matrix.

Remark 2.1. *Similar to [39, 40], a matrix G is defined as the restrictive condition of false data injection attacks, whose value is determined by attack detections. Suppose that the adversaries produce the cyber attack based on the current system state with $\zeta(t) = \zeta(x(t))$. Then, the attack signal changes with the variation of system state under the restrictive condition Eq. (3). In such a way, the adversaries are easier to escape from network security detections.*

2.3. Event-triggered mechanism

To optimize network resources further, an event-triggered mechanism similar to [1] is introduced to judge whether the current sampled data should be released or not. $t_k h$ is defined as the triggering instant while h denotes the constant sampling period. The control signal $u(t_k h)$ is holden by the ZOH until next event, which depends on the following triggering mechanism:

$$t_{k+1}h = t_k h + \min_{v\geq 1, v\in\mathbb{N}} \left\{ vh \mid e_k^T(t)\Omega e_k(t) \geq \delta x^T(t_k h)\Omega x(t_k h) \right\} \tag{4}$$

where the state error $e_k(t) \triangleq x(t_k h) - x(t_k h + vh)$, $v \in \{1, 2, \cdots, t_{k+1}h - t_k h - 1\}$, $x(t_k h + vh)$ is the current sampling data to be determined whether it should be sent out, and $x(t_k h)$ denotes the last released data. $\Omega > 0$ is a symmetric weighting matrix while $\delta \in [0, 1)$ is a prescribed index.

For network uncertainties, the transmission delay is taken into consideration. Therefore, the released data $x(t_0 h), x(t_1 h), x(t_2 h), \cdots$ will reach the controller side at the instants $(t_0 h + \tau_0), (t_1 h + \tau_1), (t_2 h + \tau_2), \cdots$. $\tau_i \in [0, \bar{\tau})$ denotes the time variant transmission delay, where $\bar{\tau} > 0$ is a prescribed value.

For the sake of analysis below, inspired by [1], we divide the interval $[t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$ into several subintervals as shown in Fig. 2. Let $\varsigma_k \triangleq t_k h + \tau_k$, $S_{k,v} \triangleq t_k h + vh + \tau_{k,v}$ and $\chi_{k,v} \triangleq [S_{k,v}, S_{k,v+1})$, then it is not difficult to prove that there always exists a constant $v_M \geq 0$ which makes $[\varsigma_k, \varsigma_{k+1}) \triangleq \cup_{v=0}^{v_M} \chi_{k,v}$.

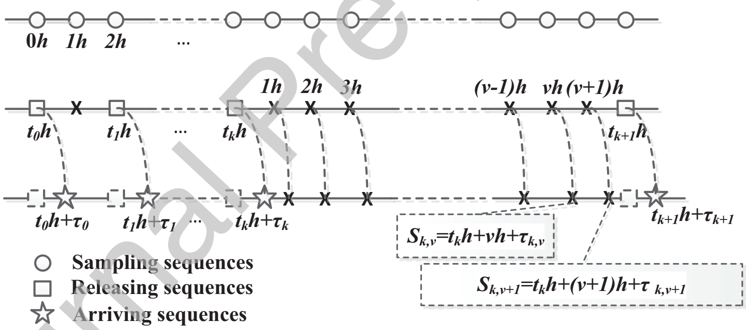

Figure 2: Time sequences of the event-triggered mechanism

Remark 2.2. *From the event-triggered mechanism (4), our method is formulated in the continuous-time domain. However, if we assume the current sampling instant is kh, then $(k + 1)h$ will be the next sampling instant. As defined, $t_0 h, t_1 h, t_2 h, \cdots$ are the releasing instants, it is easily deduced that $v_i h = t_{k+1}h - t_k h \geq h$, where $v_i h$ represents the release period. From this point of view, h can be seen as a lower bound of the release period to exclude the Zeno behavior.*

Remark 2.3. *With the development of digital sampling technology, the sampling interval is approaching to the level of 1ms, even much lower. Relatively, because of the inertia of practical systems, the system switching interval converges towards the level of 10ms. Hence, we assume that the system switching interval is always larger than the sampling interval, i.e., $h \leq (l_{q+1} - l_q)/2$, where $\{l_q\}$ are switching instants. It is of great significance to discuss the relation between the triggering instants and the switching instants.*

2.4. Co-design of the controller and ETM

In this part, we aim to establish the exact analysis model of the switched system based on the introduced event-triggered mechanism. Inspired by [1], for $t \in [\varsigma_k, \varsigma_{k+1})$, the analysis is focused on the subinterval $[S_{k,v}, S_{k,v+1})$. For $t \in [S_{k,v}, S_{k,v+1})$, defining $\eta(t) = t - t_k h - vh$ yields :

$$\eta_m \leq \tau_k \leq \eta(t) \leq h + \overline{\tau} = \eta_M \tag{5}$$

where $\eta_m \triangleq \min\{\tau_k\}$, $\overline{\tau} \triangleq \max\{\tau_k\}$.

Combining the definition of $e_k(t)$, $\eta(t)$ and Eq. (2), we have

$$u(t) = K_{\sigma(t)}(x(t - \eta(t)) + e_k(t) + \zeta(t_k h)), t \in \chi_{k,v} \tag{6}$$

For the whole interval $[0, T_f]$, the switching instants and the triggering instants are intertwined and irregular. Considering Remark 2.3, it is not difficult to find that there are two distinct situations : (i) over the time interval $\chi_{k,v}$, the switched system dwells in a subsystem; (ii) over the time interval $\chi_{k,v}$, one and only switching occurs. Then, for $t \in \chi_{k,v}$, the resulting system formulation can be expressed as two cases.

Case 2.1. *Within the interval $[S_{k,v}, S_{k,v+1}) \in [\varsigma_k, \varsigma_{k+1})$, there is no switching. Combining Eq. (6) and the system (1), we have*

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}(x(t - \eta(t)) + e_k(t) + \zeta(t_k h)) + E_{\sigma(t)}\omega(t), \qquad t \in [S_{k,v}, S_{k,v+1}) \\ x(t) = \phi(t), \qquad t \in [-\eta_M, 0] \end{cases} \tag{7}$$

where $\phi(t)$ is a continuously differentiable function on $[-\eta_M, 0]$ to supplement the initial condition of the state $x(t)$.

Case 2.2. *We assume $\varsigma_k < \cdots l_{q-1} \leqslant S_{k,v} \leqslant l_q \leqslant S_{k,v+1} \leqslant l_{q+1} \cdots \leqslant \varsigma_{k+1}$ with $\sigma(l_q) \in £$. Within the interval $[S_{k,v}, S_{k,v+1})$, there exists and only exists one switching. Then, we have*

$$\begin{cases} \dot{x}(t) = A_{\sigma(l_{q-1})}x(t) + B_{\sigma(l_{q-1})}K_{\sigma(l_{q-1})}(x(t - \eta(t)) \\ \qquad + e_k(t) + \zeta(t_k h)) + E_{\sigma(l_{q-1})}\omega(t), \qquad t \in [S_{k,v}, l_q) \\ \dot{x}(t) = A_{\sigma(l_q)}x(t) + B_{\sigma(l_q)}K_{\sigma(l_q)}(x(t - \eta(t)) \\ \qquad + e_k(t) + \zeta(t_k h)) + E_{\sigma(l_q)}\omega(t), \qquad t \in [l_q, S_{k,v+1}) \\ \qquad x(t) = \phi(t), \qquad t \in [-\eta_M, 0] \end{cases} \tag{8}$$

Remark 2.4. *As mentioned in the Introduction, [26] considers networked switched systems with the assumption that no more than one switching is permitted within $[\varsigma_k, \varsigma_{k+1})$, which deviates from practical situations. In this work, some uncertain switchings can occur over $[\varsigma_k, \varsigma_{k+1})$. Relatively speaking, it comes to the conclusion that no more than one switching can occur over $[S_{k,v}, S_{k,v+1})$ based on Remark 2.3.*

2.5. Problem statement

Before proceeding further, the following statements should be made, which will greatly contribute to the establishment of the main results.

Definition 2.1. *(Average dwell time [20]): Given a time interval* (T_1, T_2) *with* $0 \leq T_1 \leq T_2$*, let* $N_{\sigma(t)}(T_1, T_2)$ *represent the switching number over* (T_1, T_2)*, if there exist a positive constant* τ_a *and a positive number* N_0 *(the chatter bound) satisfying* $N_{\sigma(t)}(T_1, T_2) \leq N_0 + (T_2 - T_1)/\tau_a$*, we define* τ_a *as an average dwell time over* (T_1, T_2)*.*

Definition 2.2. *(Finite-time boundedness [21]): Given positive constants* c_1, c_2, T_f*, a positive definite matrix* R *and a switching signal* $\sigma(t)$*, if*

$$\max_{-\eta_M \leq \theta \leq 0} x^T(\theta) Rx(\theta) < c_1 \Rightarrow x^T(t) R(t) < c_2, \qquad t \in [0, T_f) \tag{9}$$

holds, the switched system (7) and (8) is called to be finite-time bounded.

Lemma 2.1. *([41]). Extended Wirtinger Inequality : For* $P^T = P > 0$*, the following inequality holds:*

$$\int_{a_1}^{a_2} \dot{x}^T(s) P\dot{x}(s) \geq \frac{1}{a_2 - a_1} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & 3P \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix} \tag{10}$$

where $\Omega_1 = x(a_2) - x(a_1), \Omega_2 = x(a_1) + x(a_2) - \frac{2}{a_2 - a_1} \int_{a_1}^{a_2} x(s)ds$*.*

Lemma 2.2. *([20]). Given* $x \in \mathbb{R}^n$ *and a symmetric matrix* $W \in \mathbb{R}^{n_n}$*, the following inequality holds:*

$$\lambda_{min}(W) x^T x \leq x^T W x \leq \lambda_{max}(W) x^T x \tag{11}$$

Based on the preliminary knowledge and analysis, the main purpose of this work is declared as follows: With the established system formulation (7) and (8) from the system (1), given positive system indexes c_1, c_2, T_f, positive definite matrix R, disturbance related parameter H_M and deception attack related parameter matrix G, our major target is to design applicable event-driven controllers and derive suitable constraints of the average dwell time, such that the concerned system is finite-time stable with respect to $(c_1, c_2, T_f, R, H_M, G)$.

3. Main results

In this section, sufficient conditions are proposed in Theorem 3.1 to make the system finite-time stable under cyber attacks. The design of switched sub-controllers will be presented in Theorem 3.2.

Theorem 3.1. *For given positive constants* $c_1, c_2, T_f, \eta_m, \eta_M, h, \alpha, \gamma, \delta, \mu, H_M$ *with* $\mu \geq 1$*,* $c_1 < c_2$*, and positive definite matrices* $R \in \mathbb{R}^{n_x \times n_x}$ *and* $G \in \mathbb{R}^{n_x \times n_x}$*, the switched system in (7) and (8) is finite-time bounded with respect to* $(c_1, c_2, T_f, R, H_M, G)$ *and the minimum admissible average dwell time is* τ_a^**, if there exist positive definite matrices* $P_i^l \in \mathbb{R}^{n_x \times n_x}$*,* $l \in \{1, 2, 3, 4, 5\}$*,* $\Omega \in \mathbb{R}^{n_x \times n_x}$ *and* N_i^m, M_i^m*,* $m \in \{1, 2\}$ *with appropriate dimensions such that* $\forall (i, j) \in \pounds \times \pounds, i \neq j$

$$\Pi_1^{(n)} = \begin{bmatrix} \Pi_{11} & * & * \\ \Pi_{21} & \Pi_{22} & * \\ \Pi_{31}^{(n)} & 0 & -P_i^5 \end{bmatrix} < 0, n \in \{1, 2\} \tag{12}$$

$$P_i^l < \mu P_j^l, l \in \{1, 2, 3, 4, 5\} \tag{13}$$

$$e^{\alpha T_f} < \frac{c_2\lambda_4/\lambda_2}{c_1\lambda_3(1+\varrho)/\lambda_1 + \gamma H_M} \tag{14}$$

$$\tau_a \geq \tau_a^* = \frac{T_f \ln\mu}{\ln(c_2\lambda_4/\lambda_2) - \ln(c_1\lambda_3(1+\varrho)/\lambda_1 + \gamma H_M) - \alpha T_f} \tag{15}$$

where

$$\Pi_{11} = \begin{bmatrix} \Xi_1 & * & * & * & * & * & * & * \\ -2P_i^4 & \Xi_2 & * & * & * & * & * & * \\ 6P_i^4 & 6P_i^4 & -12P_i^4 & * & * & * & * & * \\ 0 & 0 & 0 & \Xi_3 & * & * & * & \\ K_i^T B_i^T P_i^1 & \Xi_4 & 0 & \Xi_5 & \Xi_6 & * & * & * \\ K_i^T B_i^T P_i^1 & 0 & 0 & 0 & 0 & -\Omega & * & * \\ E_i^T P_i^1 & 0 & 0 & 0 & 0 & 0 & -\gamma & * \\ K_i^T B_i^T P_i^1 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Pi_{21} = \begin{bmatrix} \eta_m A_i & 0 & 0 & 0 & \eta_m B_i K_i & \eta_m B_i K_i & \eta_m E_i & \eta_m B_i K_i \\ \beta A_i & 0 & 0 & 0 & \beta B_i K_i & \beta B_i K_i & \beta E_i & \beta B_i K_i \\ 0 & 0 & 0 & 0 & \sqrt{\sigma} & \sqrt{\sigma} & 0 & 0 \\ 0 & 0 & 0 & 0 & G & G & 0 & 0 \end{bmatrix},$$

$$\Pi_{31}^{(1)} = \begin{bmatrix} 0 & \beta N_i^{1^T} & 0 & 0 & \beta N_i^{2^T} & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{31}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & \beta M_i^{1^T} & \beta M_i^{2^T} & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{22} = diag\{-P_i^{4^{-1}}, -P_i^{5^{-1}}, -\Omega^{-1}, -I\},$$

$$\Xi_1 = A_i^T P_i^1 + P_i^1 A_i + P_i^2 + P_i^3 - \alpha P_i^1 - 4P_i^4, \Xi_2 = -4P_i^4 - e^{\alpha\eta_m} P_i^2 + N_i^1 + N_i^{1^T},$$

$$\Xi_3 = -e^{\alpha\eta_m} P_i^3 - M_i^2 - M_i^{2^T}, \Xi_4 = -N_i^2 - N_i^{1^T}, \Xi_5 = M_i^{2^T} - M_i^1, \Xi_6 = -N_i^2 - N_i^{2^T} + M_i^1 + M_i^{1^T},$$

$$\beta = \sqrt{\eta_M - \eta_m}, \lambda_1 = \lambda_{min}(R), \lambda_2 = \lambda_{max}(R),$$

$$\lambda_3 = min\{\lambda_{min}(P_i^l)\}, \lambda_4 = max\{\lambda_{max}(P_i^l)\}, (l = 1,2,3,4,5),$$

$$\varrho = -\frac{1}{\alpha}\left(2 - e^{\alpha\eta_m} - e^{\alpha\eta_M} + \eta_m^2 + \eta_M - \eta_m + \frac{\eta_m}{\alpha}(1 - e^{\alpha\eta_m}) + \frac{1}{\alpha}(e^{\alpha\eta_m} - e^{\alpha\eta_M})\right)$$

Proof. Construct the following Lyapunov-Krasovskii functional candidate

$$V_{\sigma(t)}(t) = V_{\sigma(t),1}(t) + V_{\sigma(t),2}(t) + V_{\sigma(t),3}(t) + V_{\sigma(t),4}(t) + V_{\sigma(t),5}(t) \tag{16}$$

where

$$V_{\sigma(t),1}(t) = x^T(t) P_{\sigma(t)}^1 x(t)$$

$$V_{\sigma(t),2}(t) = \int_{t-\eta_m}^{t} e^{\alpha(t-s)} x^T(s) P_{\sigma(t)}^2 x(s) ds$$

$$V_{\sigma(t),3}(t) = \int_{t-\eta_M}^{t} e^{\alpha(t-s)} x^T(s) P_{\sigma(t)}^3 x(s) ds$$

$$V_{\sigma(t),4}(t) = \eta_m \int_{t-\eta_m}^{t} \int_{v}^{t} e^{\alpha(t-s)} \dot{x}^T(s) P_{\sigma(t)}^4 \dot{x}(s) ds dv$$

$$V_{\sigma(t),5}(t) = \int_{t-\eta_M}^{t-\eta_m} \int_v^t e^{\alpha(t-s)} \dot{x}^T(s) P^5_{\sigma(t)} \dot{x}(s) ds dv$$

Case 3.1. *Consider the system (7), that is, within every interval $[S_{k,v}, S_{k,v+1}) \in [\varsigma_k, \varsigma_{k+1})$, there is no switching. Suppose that only the subsystem mode $i \in \pounds$ is active in the interval $[S_{k,v}, S_{k,v+1})$.*

Then, for $t \in [S_{k,v}, S_{k,v+1})$, taking the derivative on $V_{i,l}(t)$ for $(l = 1, 2, 3, 4, 5)$, we have

$$\begin{aligned}
\dot{V}_i(t) =&\alpha V_i(t) + \gamma\omega^T(t)\omega(t) + x^T(t)(A_i^T P_i^1 + P_i^1 A_i + P_i^2 + P_i^3 - \alpha P_i^1)x(t) + 2x^T(t)E_i^T P_i^1 \omega(t)) \\
&+ 2x^T(t)K_i^T B_i^T P_i^1 x(t-\eta(t)) + 2x^T(t)K_i^T B_i^T P_i^1 e_k(t) + 2x^T(t)K_i^T B_i^T P_i^1 \zeta(t_k h) \\
&- e^{\alpha\eta_m} x^T(t-\eta_m)P_i^2 x(t-\eta_m) - e^{\alpha\eta_M} x^T(t-\eta_M)P_i^3 x(t-\eta_M) + \eta_m^2 \dot{x}^T(t)P_i^4 \dot{x}(t) \\
&+ (\eta_M - \eta_m)\dot{x}^T(t)P_i^5 \dot{x}(t) - \eta_m \int_{t-\eta_m}^t e^{\alpha(t-s)} \dot{x}^T(s)P_i^4 \dot{x}(s)ds - \int_{t-\eta_M}^{t-\eta_m} e^{\alpha(t-s)} \dot{x}^T(s)P_i^5 \dot{x}(s)ds \\
&+ 2\xi^T(t)N_i \left[x(t-\eta_m) - x(t-\eta(t)) - \int_{t-\eta(t)}^{t-\eta_m} \dot{x}(s)ds \right] \\
&+ 2\xi^T(t)M_i \left[x(t-\eta(t)) - x(t-\eta_M)) - \int_{t-\eta_M}^{t-\eta(t)} \dot{x}(s)ds \right] \\
&- \gamma\omega^T(t)\omega(t) + e_k^T(t)\Omega e_k(t) - e_k^T(t)\Omega e_k(t) + \zeta^T(t_k h)\zeta(t_k h) - \zeta^T(t_k h)\zeta(t_k h)
\end{aligned} \tag{17}$$

where $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-\eta_m) & \frac{1}{\eta_m}\int_{t-\eta_m}^t x^T(s)ds & x^T(t-\eta_M) & x^T(t-\eta(t)) & e_k^T(t) & \omega^T(t) & \zeta^T(t_k h) \end{bmatrix}^T$, $N_i = \begin{bmatrix} 0 & N_i^{1^T} & 0 & 0 & N_i^{2^T} & 0 & 0 & 0 \end{bmatrix}^T$, *and* $M_i = \begin{bmatrix} 0 & 0 & 0 & M_i^{1^T} & M_i^{2^T} & 0 & 0 & 0 \end{bmatrix}^T$.

According to Lemma 2.1, we have

$$\begin{aligned}
-\eta_m \int_{t-\eta_m}^t e^{\alpha(t-s)} \dot{x}^T(s)P_i^4 \dot{x}(s)ds &\leq -\eta_m \int_{t-\eta_m}^t \dot{x}^T(s)P_i^4 \dot{x}(s)ds \\
&\leq - \begin{bmatrix} \xi(t)T_1 \\ \xi(t)T_2 \end{bmatrix}^T \begin{bmatrix} P_i^4 & 0 \\ 0 & 3P_i^4 \end{bmatrix} \begin{bmatrix} \xi(t)T_1 \\ \xi(t)T_2 \end{bmatrix}
\end{aligned} \tag{18}$$

where $T_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, T_2 = \begin{bmatrix} I & I & -2I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

For matrices N_i, M_i and $P_i^5 > 0$, we notice that

$$-2\xi^T(t)N_i \int_{t-\eta(t)}^{t-\eta_m} \dot{x}(s)ds \leq (\eta(t) - \eta_m)\xi^T(t)N_i P_i^{5^{-1}} N_i^T \xi(t) + \int_{t-\eta(t)}^{t-\eta_m} \dot{x}(s)P_i^5 \dot{x}(s)ds \tag{19}$$

$$-2\xi^T(t)M_i \int_{t-\eta_M}^{t-\eta(t)} \dot{x}(s)ds \leq (\eta_M - \eta(t))\xi^T(t)M_i P_i^{5^{-1}} M_i^T \xi(t) + \int_{t-\eta_M}^{t-\eta(t)} \dot{x}(s)P_i^5 \dot{x}(s)ds \tag{20}$$

Combining the ETM in (4), the restricted condition of cyber attacks in Eq. (3) and Eq. (17)-Eq.(20), we have

$$\begin{aligned}
&\dot{V}(t) - \alpha V(t) - \gamma\omega^T(t)\omega(t) \\
&\leq \xi^T(t) \left(\Pi_{11} - \Pi_{21}^T \Pi_{22}^{-1} \Pi_{21} + (\eta(t) - \eta_m)N_i P_i^{5^{-1}} N_i^T + (\eta_M - \eta(t))M_i P_i^{5^{-1}} M_i^T \right) \xi(t)
\end{aligned} \tag{21}$$

Based on the sufficient condition (12), the following conditions are derived by using the Schur complement

$$\Pi_{11} - \Pi_{21}^T \Pi_{22}^{-1} \Pi_{21} + (\eta_M - \eta_m) N_i P_i^{5^{-1}} N_i^T \leq 0 \tag{22}$$

$$\Pi_{11} - \Pi_{21}^T \Pi_{22}^{-1} \Pi_{21} + (\eta_M - \eta_m) M_i P_i^{5^{-1}} M_i^T \leq 0 \tag{23}$$

Inspired by [42], one has

$$\dot{V}(t) \leq \alpha V(t) + \gamma \omega^T(t)\omega(t), \forall t \in [S_{k,v}, S_{k,v+1}) \tag{24}$$

Integrating both sides of (24) from $S_{k,v}$ to t, we have

$$V_i(t) \leq e^{\alpha(t-S_{k,v})} V_i(x(S_{k,v})) + \gamma \int_{S_{k,v}}^{t} e^{\alpha(t-s)} \omega^T(s)\omega(s)ds, \forall t \in [S_{k,v}, S_{k,v+1}) \tag{25}$$

Case 3.2. *Consider the system (8), there always exsits an interval $[S_{k,v}, S_{k,v+1}) \in [\varsigma_k, \varsigma_{k+1})$, which includes one switching. We assume $l_i < S_{k,v} < l_j < S_{k,v+1}$, where l_i, l_j are switching instants.*

For $t \in [S_{k,v}, l_j)$, similar to the proof in Case 3.1, it follows from (25) that

$$V_{\sigma(l_i)}(t) \leq e^{\alpha(t-S_{k,v})} V_{\sigma(l_i)}(x(S_{k,v})) + \gamma \int_{S_{k,v}}^{t} e^{\alpha(t-s)} \omega^T(s)\omega(s)ds, \forall t \in [S_{k,v}, l_j) \tag{26}$$

For $t \in [l_j, S_{k,v+1})$, considering the sufficient condition (13) yields

$$V_{\sigma(l_j)}(t) \leq \mu V_{\sigma(l_i)}(t) \leq \mu e^{\alpha(t-S_{k,v})} V_{\sigma(l_i)}(x(S_{k,v})) + \mu\gamma \int_{S_{k,v}}^{t} e^{\alpha(t-s)} \omega^T(s)\omega(s)ds, \forall t \in [l_j, S_{k,v+1}) \tag{27}$$

Taking together the analysis in Case 3.1 and Case 3.2, we can gain the result in the whole interval $[0, T_f]$. According to Definition 2.1, we let N_{sn} be the switching number over $[0, T_f]$ and $\{l_1, l_2, \cdots, l_{N_{sn}}\}$ be the corresponding switching instants. Meanwhile, we define $S_{\Psi(l_q),\Gamma(l_q)}$ to be the initial instant of the minimum subinterval at the right of the switching instant l_q. The two functions $\Psi(l_q)$ and $\Gamma(l_q)$ are used to label the exact position on the release intervals and its subintervals with $l_q \in [\varsigma_{\Psi(l_q)}, \varsigma_{\Psi(l_q)+1})$ and $l_q \in [S_{\Psi(l_q),\Gamma(l_q)-1}, S_{\Psi(l_q),\Gamma(l_q)})$. Moreover, we define $S^-_{\Psi(l_q),\Gamma(l_q)}$ as the sampling instant closely before $S_{\Psi(l_q),\Gamma(l_q)}$, which yields $0 \leqslant S^-_{\Psi(l_1),\Gamma(l_1)} < l_1 \leqslant S_{\Psi(l_1),\Gamma(l_1)} \leqslant S^-_{\Psi(l_2),\Gamma(l_2)} < l_2 \leqslant S_{\Psi(l_2),\Gamma(l_2)} < \cdots < S^-_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})} < l_{N_{sn}} \leqslant S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})} \leqslant T_f$.

Taking advantage of the continuity of $V_{\sigma(t)}(t)$ on each subsystem, combining Eq. (25) - Eq. (27) and the sufficient condition (13), $\forall t \in [S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}, T_f)$, we have

$$
\begin{aligned}
V_{\sigma(t)}(t) &\leq e^{\alpha(t-S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}V_{\sigma(l_{N_{sn}})}(x(S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}))\\
&+\gamma e^{\alpha(t-S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}\int_{S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}^{t}\omega^T(s)\omega(s)ds\\
&\leq e^{\alpha(t-S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}[\mu e^{\alpha(S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}-S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}\\
&V_{\sigma(l_{N_{sn}-1})}(x(S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}))+\mu\gamma\int_{S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}^{S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}e^{\alpha(t-S)}\omega^T(s)\omega(s)ds]\\
&+\gamma e^{\alpha(t-S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}\int_{S_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}^{t}\omega^T(s)\omega(s)ds\\
&\leq \mu e^{\alpha(t-S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}V_{\sigma(l_{N_{sn}-1})}(x(S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}))\\
&+\mu\gamma e^{\alpha(t-S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}\int_{S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}^{t}\omega^T(s)\omega(s)ds\\
&\leq \mu e^{\alpha(t-S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}[e^{\alpha(S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}-S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})})}\\
&V_{\sigma(l_{N_{sn}-1})}(x(S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})}))+\gamma\int_{S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})}}^{S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}e^{\alpha(t-S)}\omega^T(s)\omega(s)ds]\\
&+\mu\gamma e^{\alpha(t-S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})})}\int_{S^{-}_{\Psi(l_{N_{sn}}),\Gamma(l_{N_{sn}})}}^{t}\omega^T(s)\omega(s)ds\\
&\leq \mu e^{\alpha(t-S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})})}V_{\sigma(l_{N_{sn}-1})}(x(S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})}))\\
&+\mu\gamma e^{\alpha(t-S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})})}\int_{S_{\Psi(l_{N_{sn}-1}),\Gamma(l_{N_{sn}-1})}}^{t}\omega^T(s)\omega(s)ds\\
&\vdots\\
&\leq \mu^{N_{sn}}e^{\alpha t}V_{\sigma(0)}(x(0))\\
&+\mu^{N_{sn}}\gamma e^{\alpha t}\int_0^t\omega^T(s)\omega(s)ds
\end{aligned}
\tag{28}
$$

According to Lemma 2.2 and the constructed Lyapunov-Krasovskii functional candidate (16), we can notice that

$$
V_{\sigma(0)}(0) \leq \frac{\lambda_3}{\lambda_1}c_1(1+\varrho) \tag{29}
$$

Combining Eq. (28) - Eq. (29) and Assumption 2.1, we have

$$
\begin{aligned}
x^T(t)Rx(t) \leq \frac{\lambda_2}{\lambda_4}V_{\sigma(t)}(t) &\leq \frac{\lambda_2}{\lambda_4}\mu^{N_{sn}}e^{\alpha T_f}(V_{\sigma(0)}(0)+\gamma\int_0^{T_f}\omega^T(s)\omega(s))ds\\
&\leq \frac{\lambda_2}{\lambda_4}\mu^{N_{sn}}e^{\alpha T_f}(\frac{\lambda_3}{\lambda_1}c_1(1+\varrho)+\gamma H_M
\end{aligned}
\tag{30}
$$

If $\mu = 1$, from the sufficient condition (14), one has

$$x^T(t)Rx(t) \leq \frac{\lambda_2}{\lambda_4} e^{\alpha T_f} (\frac{\lambda_3}{\lambda_1} c_1(1+\varrho) + \gamma H_M < c_2 \tag{31}$$

If $\mu > 1$, from the sufficient condition (15) and the defined average dwell time in Definition 2.1, we have

$$\frac{T_f}{\tau_a} \leq \frac{\ln(c_2\lambda_4/\lambda_2) - \ln(c_1\lambda_3(1+\varrho)/\lambda_1 + \gamma H_M) - \alpha T_f}{\ln \mu} \tag{32}$$

Substituting (32) into (30) yields

$$x^T(t)Rx(t) \leq \frac{\lambda_2\lambda_3 c_1(1+\varrho) + \lambda_1\lambda_2\gamma H_M}{\lambda_1\lambda_4} e^{\alpha T_f} \frac{c_2\lambda_1\lambda_4}{\lambda_2\lambda_3 c_1(1+\varrho) + \lambda_1\lambda_2\gamma H_M} e^{-\alpha T_f} < c_2 \tag{33}$$

The proof is completed.

Remark 3.1. *Throughout the whole proof, it is divided into Case 3.1 and Case 3.2, which are corresponding to the system formulation (7) and (8). In Case 3.2, $\forall t \in [l_j, S_{k,v+1})$, it comes to the similar preliminary result as in Case 3.1 through some special mathematical techniques. It also explains the necessity of the system formulation (7) and (8) from the point of mathematics.*

On the basis of Theorem 3.1, the design of the sub-controller gains $K_{\sigma(t)}$ will be presented in Theorem 3.2.

Theorem 3.2. *For given positive constants $c_1, c_2, T_f, \eta_m, \eta_M, h, \alpha, \gamma, \delta, \mu, H_M, \rho_i^l (l = 1, 2, 3, 4)$ with $\mu \geq 1$, $c_1 < c_2$, and positive definite matrices $R \in \mathbb{R}^{n_x \times n_x}$ and $G \in \mathbb{R}^{n_x \times n_x}$, the control loop (7) and (8) is finite-time bounded with respect to $(c_1, c_2, T_f, R, H_M, G)$ with the controller gain $K_i = Y_i X_i^{-1}$, if there exist positive definite matrices $X_i \in \mathbb{R}^{n_x \times n_x}$, $\widetilde{P}_i^l \in \mathbb{R}^{n_x \times n_x}$, $l \in \{2, 3, 4, 5\}$, $\widetilde{\Omega} \in \mathbb{R}^{n_x \times n_x}$ and $Y_i, \widetilde{N}_i^m, \widetilde{M}_i^m$, $m \in \{1, 2\}$ with appropriate dimensions such that $\forall (i, j) \in \pounds \times \pounds, i \neq j$*

$$\widetilde{\Pi}_1^{(n)} = \begin{bmatrix} \widetilde{\Pi}_{11} & * & * \\ \widetilde{\Pi}_{21} & \widetilde{\Pi}_{22} & * \\ \widetilde{\Pi}_{31}^{(n)} & 0 & -\widetilde{P}_i^5 \end{bmatrix} < 0, n \in \{1, 2\} \tag{34}$$

$$X_i < \mu X_j, \widetilde{P}_i^l < \mu \widetilde{P}_j^l, l \in \{2, 3, 4, 5\} \tag{35}$$

$$e^{\alpha T_f} < \frac{c_2\lambda_4/\lambda_2}{c_1\lambda_3(1+\varrho)/\lambda_1 + \gamma H_M} \tag{36}$$

$$\tau_a \geq \tau_a^* = \frac{T_f \ln \mu}{\ln(c_2\lambda_4/\lambda_2) - \ln(c_1\lambda_3(1+\varrho)/\lambda_1 + \gamma H_M) - \alpha T_f} \tag{37}$$

where

$$\widetilde{\Pi}_{11} = \begin{bmatrix} \widetilde{\Xi}_1 & * & * & * & * & * & * & * \\ -2\widetilde{P}_i^4 & \widetilde{\Xi}_2 & * & * & * & * & * & * \\ 6\widetilde{P}_i^4 & 6\widetilde{P}_i^4 & -12\widetilde{P}_i^4 & * & * & * & * & * \\ 0 & 0 & 0 & \widetilde{\Xi}_3 & * & * & * & \\ Y_i^T B_i^T & \widetilde{\Xi}_4 & 0 & \widetilde{\Xi}_5 & \widetilde{\Xi}_6 & * & * & * \\ Y_i^T B_i^T & 0 & 0 & 0 & 0 & -\widetilde{\Omega} & * & * \\ E_i^T & 0 & 0 & 0 & 0 & 0 & -\gamma I & * \\ Y_i^T B_i^T & 0 & 0 & 0 & 0 & 0 & 0 & -2\rho_i^1 X + \rho_i^1\rho_i^1 I \end{bmatrix},$$

$$\widetilde{\Pi}_{21} = \begin{bmatrix} \eta_m A_i X_i & 0 & 0 & 0 & \eta_m B_i Y_i & \eta_m B_i Y_i & \eta_m E_i & \eta_m B_i Y_i \\ \beta A_i X_i & 0 & 0 & 0 & \beta B_i Y_i & \beta B_i Y_i & \beta E_i & \beta B_i Y_i \\ 0 & 0 & 0 & 0 & \sqrt{\sigma} X_i & \sqrt{\sigma} X_i & 0 & 0 \\ 0 & 0 & 0 & 0 & G X_i & G X_i & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{31}^{(1)} = \begin{bmatrix} 0 & \beta\widetilde{N}_i^{1^T} & 0 & 0 & \beta\widetilde{N}_i^{2^T} & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{31}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & \beta\widetilde{M}_i^{1^T} & \beta\widetilde{M}_i^{2^T} & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Pi}_{22} = diag\{-2\rho_i^2 X_i + \rho_i^2\rho_i^2\widetilde{P}_i^4, -2\rho_i^3 X_i + \rho_i^3\rho_i^3\widetilde{P}_i^5, -2\rho_i^4 X_i + \rho_i^4\rho_i^4\widetilde{\Omega}, -I\},$$

$$\widetilde{\Xi}_1 = X_i A_i^T + A_i X_i + \widetilde{P}_i^2 + \widetilde{P}_i^3 - \alpha X_i - 4\widetilde{P}_i^4, \widetilde{\Xi}_2 = -4\widetilde{P}_i^4 - e^{\alpha\eta_m}\widetilde{P}_i^2 + \widetilde{N}_i^1 + \widetilde{N}_i^{1^T},$$

$$\widetilde{\Xi}_3 = -e^{\alpha\eta_m}\widetilde{P}_i^3 - \widetilde{M}_i^2 - \widetilde{M}_i^{2^T}, \widetilde{\Xi}_4 = -\widetilde{N}_i^2 - \widetilde{N}_i^{1^T}, \widetilde{\Xi}_5 = \widetilde{M}_i^{2^T} - \widetilde{M}_i^1, \widetilde{\Xi}_6 = -\widetilde{N}_i^2 - \widetilde{N}_i^{2^T} + \widetilde{M}_i^1 + \widetilde{M}_i^{1^T},$$

$$\beta = \sqrt{\eta_M - \eta_m}, \lambda_1 = \lambda_{min}(R), \lambda_2 = \lambda_{max}(R),$$

$$\lambda_3 = min\{1/\lambda_{max}(X_i), \lambda_{min}(\widetilde{P}_i^l)\}, \lambda_4 = max\{1/\lambda_{min}(X_i), \lambda_{max}(\widetilde{P}_i^l)\}, (l = 2, 3, 4, 5),$$

$$\varrho = -\frac{1}{\alpha}\left(2 - e^{\alpha\eta_m} - e^{\alpha\eta_M} + \eta_m^2 + \eta_M - \eta_m + \frac{\eta_m}{\alpha}(1 - e^{\alpha\eta_m}) + \frac{1}{\alpha}(e^{\alpha\eta_m} - e^{\alpha\eta_M})\right)$$

Proof. Define $X_i = P_i^{1^{-1}}, \widetilde{P}_i^l = X_i P_i^l X_i, l \in \{2, 3, 4, 5\}, \widetilde{\Omega} = X_i \Omega X_i, \widetilde{N}_i^m = X_i N_i^m X_i, \widetilde{M}_i^m = X_i M_i^m X_i$, $m \in \{1, 2\}$, and $Y_i = K_i X_i$.

Notice that $(\rho\Lambda - Z)\Lambda^{-1}(\rho\Lambda - Z) \geq 0$ holds for $\Lambda > 0, Z > 0$ and $\rho > 0$. It is equivalent to

$$-Z\Lambda Z \leq -2\rho Z + \rho^2\Lambda \tag{38}$$

Let $\Lambda = \Omega^{-1}$, $Z = P_i^1$ and $\rho = \rho_i^4$ we have

$$-P_i^1\Omega^{-1}P_i^1 \leq -2\rho_i^4 P_i^1 + \rho_i^4\rho_i^4\Omega \tag{39}$$

Similary,

$$-P_i^1 P_i^{4^{-1}} P_i^1 \leq -2\rho_i^2 P_i^1 + \rho_i^2\rho_i^2 P_i^4 \tag{40}$$

$$-P_i^1 P_i^{5^{-1}} P_i^1 \leq -2\rho_i^3 P_i^1 + \rho_i^3\rho_i^3 P_i^5 \tag{41}$$

$$-X_i X_i \leq -2\rho_i^1 X_i + \rho_i^1\rho_i^1 I \tag{42}$$

By pre- and post-multiplying (12) with diag$\{I, I, I, I, I, I, I, I, P_i^1, P_i^1, P_i^1, I, I\}$ and theirs transposes, pre- and post-multiplying with diag$\{X, X, X, X, X, X, I, X, X, X, X, I, X\}$ and theirs transposes, combining (39) - (42), it indicates that (12) is a sufficient condition for (34). Meanwhile, by the appropriate transformations, it is not difficult to see (13) is a sufficient condition for (35). That completes the proof.

4. Illustrative example

In this section, the effectiveness of the proposed event-triggered communication scheme and the theoretical results is demonstrated by an illustrative example.

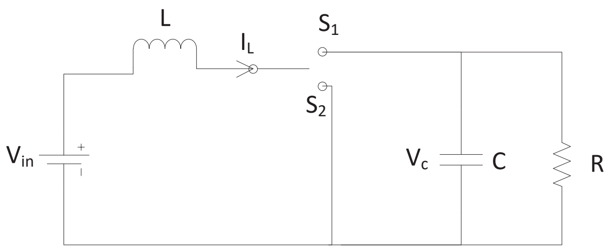

Figure 3: The boost converter circuit

A boost converter circuit in Fig. 3 is modeled with two switching modes (S_1, S_2). The system matrices are described as follows:

$$S_1 : A_1 = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/(RC) \end{bmatrix}, B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \tag{43}$$

$$S_2 : A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -1/(RC) \end{bmatrix}, B_2 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \tag{44}$$

Define the state variable $x(t) = [I_L, V_C]^T$ and the control input $u(t) = V_{in}$, where V_{in} is assumed to be an alternating voltage. The remaining system matrices are

$$E_1 = \begin{bmatrix} -1.5 \\ 2 \end{bmatrix}, E_2 = \begin{bmatrix} -1.3 \\ 2 \end{bmatrix} \tag{45}$$

In this work, we tend to assume the switched system unstable without feedback control, then the system parameters are set as :$L = 0.1H, R = 20\Omega, C = 5F$. Suppose that $c_1 = 0.02, c_2 = 20, T_f = 2s, h = 10ms, \eta_M = 9ms, \eta_m = 3ms, \alpha = 0.8, \mu = 1.01, \gamma = 4, R = I_{2\times2}$. For a predefined $\omega(t) = 0.3/(1+t)$, according to Assumption 2.1, we choose $H_M = 0.06$. Inspired by [39], the function of the cyber attack $\zeta(t)$ is supposed as $\zeta(t) = \begin{bmatrix} -tanh(0.01x_2(t)) \\ -tanh(0.01x_1(t)) \end{bmatrix}$. According to Assumption 2.3, the attack related parameter $G = diag\{0.01, 0.01\}$ can be derived easily.

With the given parameters, our objective is to gain the maximum δ possible and the minimum τ_a^* possible, and obtain a set of sub-controller gains $K_{\sigma(t)}$, such that the boost converter circuit system is finite-time stable.

By using LMI control toolbox, the maximum value of δ satisfying Theorem 3.2 is obtained, $\delta = 0.6$, with $\rho_1^1 = 0.1, \rho_2^1 = 0.05, \rho_1^2 = \rho_2^2 = \rho_1^3 = \rho_2^3 = \rho_1^4 = \rho_2^4 = 1.5$. Then, we choose two representative values of δ, 0.6 and 0.05, as two cases to illustrate the output responses.

Case 4.1. $\delta = 0.6$. *The corresponding controller gains and the weight of event-triggering condition are given by*

$$K_1 = \begin{bmatrix} -0.0515 & 0.2717 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0767 & -0.6512 \end{bmatrix} \tag{46}$$

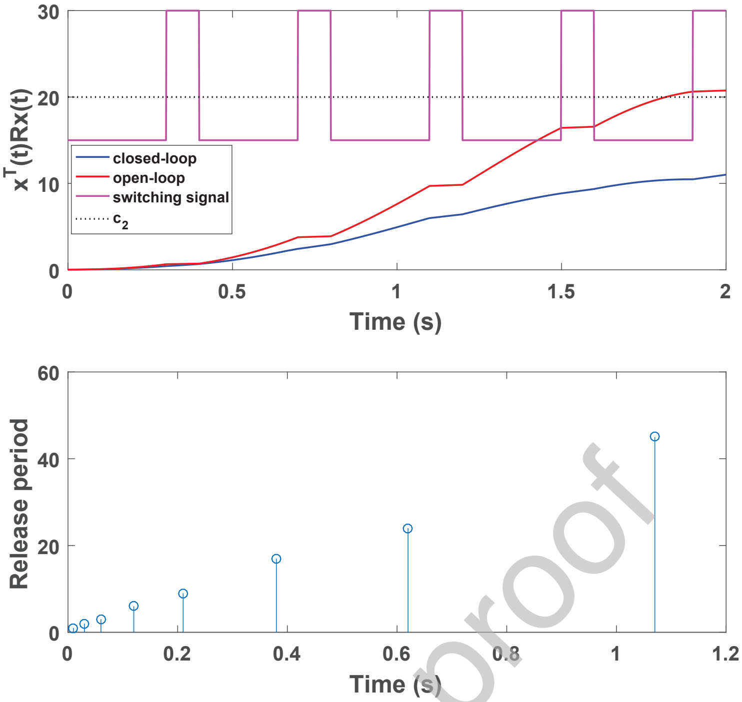

Figure 4: Trajectoriy of $x^T(t)Rx(t)$ in the open-loop and the closed-loop switched system and triggering instants

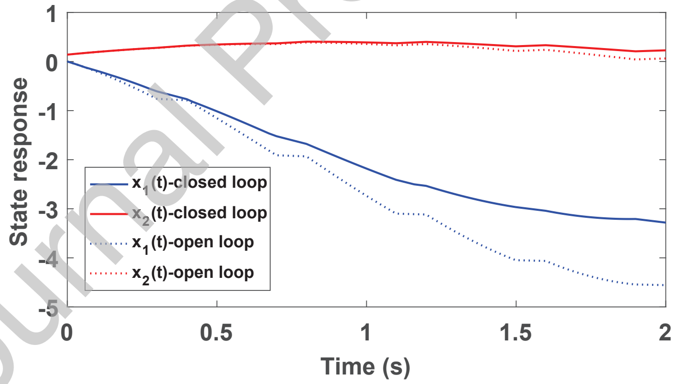

Figure 5: Trajectoriy of $x(t)$ in the open-loop and the closed-loop switched system

$$\Omega = \begin{bmatrix} 5.6428 & -0.1363 \\ -0.1363 & 0.14 \end{bmatrix} \tag{47}$$

From (37), we have $\tau_a^* = 0.2106s$. The output responses are depicted in Fig. 4 - 6 with the initial state $[0, 0.14]^T$. At most 9 switchings are permitted in the interval $[0, 2s]$. A comparison is made of the trajectories of $x^T(t)Rx(t)$ between the open-loop and the closed-loop switched system in Fig. 4. We can notice that the value of $x^T(t)Rx(t)$ in the open-loop control exceeds the assigned index $c_2 = 20$ while the closed-loop maintains in an acceptable level. It is clearly indicated that the designed event-driven controllers can ensure the boost converter circuit system FTB under cyber attacks. Meanwhile, only 8 sampling data are released comparing to the total number of

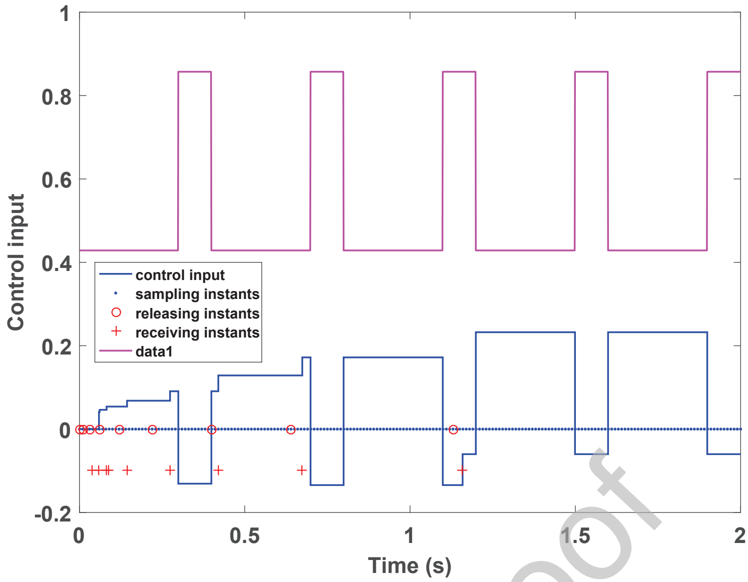

Figure 6: Response of the control input

sampling data $N_{sn} = T_f/h = 200$, which makes it clear the utilization of network resources are indeed improved. Fig. 5 illustrates the corresponding trajectories of system states and Fig. 6 describes the corresponding control input. Every jump of the control input is consistent with the switching instant or the receiving instant over time-varying networks.

Case 4.2. $\delta = 0.05$. *The corresponding controller gains and the weight of event-triggering condition are given by*

$$K_1 = \begin{bmatrix} -0.0681 & 0.3641 \end{bmatrix}, K_2 = \begin{bmatrix} -0.1032 & -0.7965 \end{bmatrix} \tag{48}$$

$$\Omega = \begin{bmatrix} 5.8719 & -0.1417 \\ -0.1417 & 0.1448 \end{bmatrix} \tag{49}$$

From (37), we have $\tau_a^* = 0.2082s$. For the convenience of comparison, the same switching signal is chosen as in Case 4.1. Correspondingly, the output responses are illustrated in Fig. 7 - 9. On the whole, a similar conclusion can be drawn as in Case 4.1, i.e., the boost converter circuit system is guaranteed to be FTB. Simultaneously, there still exist differences between Case 4.1 and Case 4.2. With the value of δ changing from 0.6 to 0.05, the releasing number increases from 9 to 20 as shown in Fig. 7. It is confirmed that the triggering threshold δ determines the trigger frequency. Meanwhile, from the perspective of control effect, Case 4.2 illustrates a higher control level with a smaller value of $x^T(t)Rx(t)$, which can also be demonstrated by the system state in Fig. 8. Of course, $\delta = 0.6$ is satisfactory for the prescribed bound $c_2 = 20$, which saves more communication resources.

5. Conclusion

On the basis of the event-triggered mechanism, this paper has investigated the finite-time control problem for networked switched systems. By the specific design of physical plant, the syn-

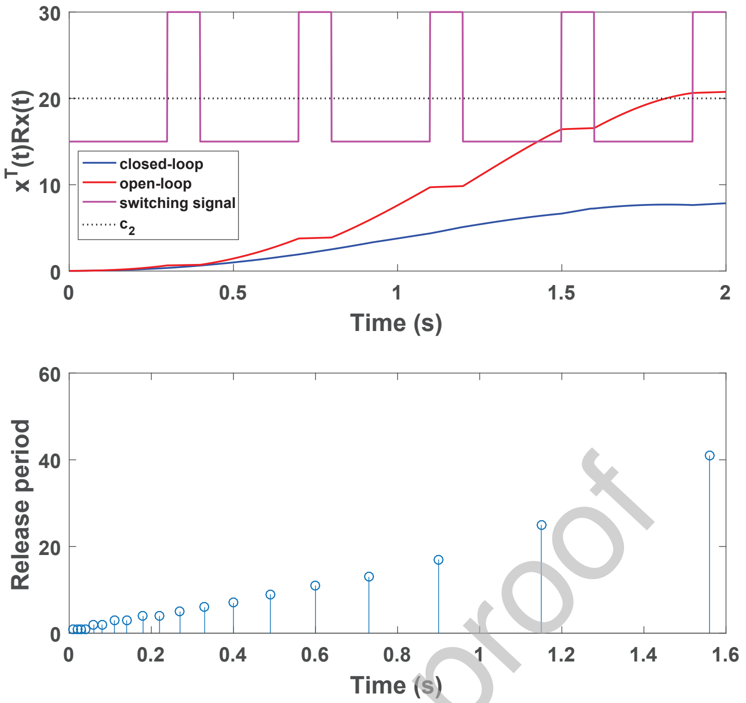

Figure 7: Trajectoriy of $x^T(t)Rx(t)$ in the open-loop and the closed-loop switched system and triggering instants

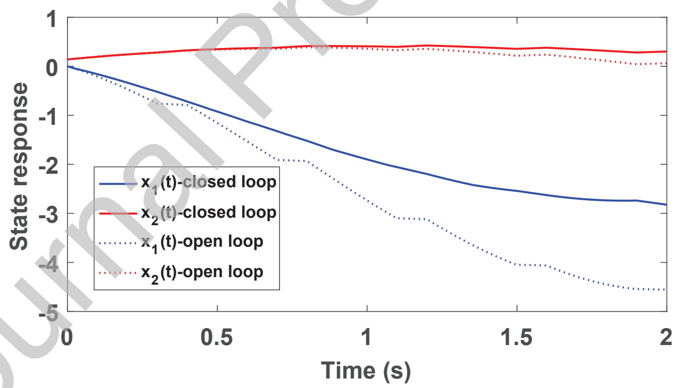

Figure 8: Trajectoriy of $x(t)$ in the open-loop and the closed-loop switched system

chronism between the switched system and its corresponding controllers is guaranteed. Sufficient conditions for finite-time boundedness have been developed under network-induced delays and cyber attacks. Then, the design methods of switched sub-controllers, optimal event-triggered parameters and restrictive average dwell-time switching law are presented. A boost converter circuit has been introduced as an illustrative example. By using the LMI technique, appropriate system parameters can be gained. Through detailed comparative analysis, it is demonstrated that the presented results can not only contribute to optimize limited network resources but also improve the system security level.

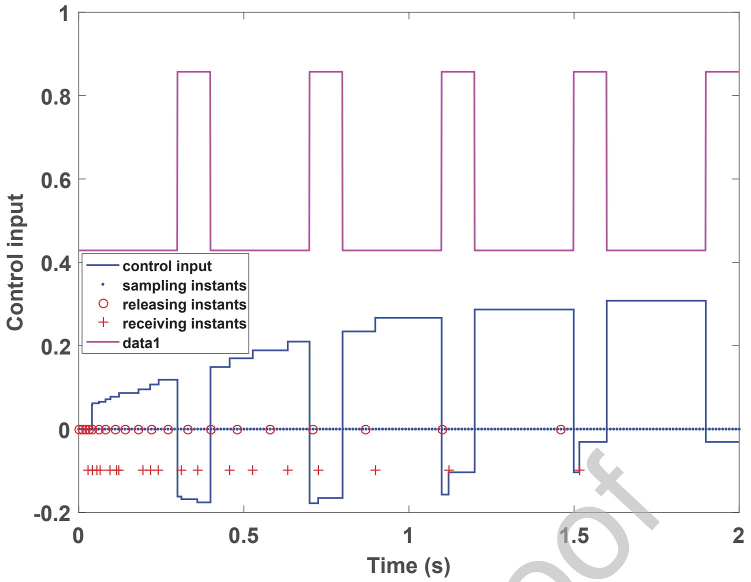

Figure 9: Response of the control input

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References

[1] D. Yue, E. Tian, and Q. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.

[2] B. L. Zhang and Q.-L. Han, "Network-based modelling and active control for offshore steel jacket platform with tmd mechanisms," *Journal of Sound and Vibration*, vol. 333, no. 25, pp. 6796–6814, 2014.

[3] H. Wang, D. Zhang, and R. Lu, "Event-triggered H_∞ filter design for markovian jump systems with quantization," *Nonlinear Analysis: Hybrid Systems*, vol. 28, pp. 23 – 41, 2018.

[4] Z. Gu, T. Zhang, F. Yang, H. Zhao, and M. Shen, "A novel event-triggered mechanism for networked cascade control system with stochastic nonlinearities and actuator failures," *Journal of the Franklin Institute*, vol. 356, no. 4, pp. 1955 – 1974, 2019.

[5] C. Peng, Q. Han, and D. Yue, "Communication-delay-distribution-dependent decentralized control for large-scale systems with IP-based communication networks," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 3, pp. 820–830, 2013.

[6] W. Zou and Z. Xiang, "Event-triggered distributed containment control of heterogeneous linear multi-agent systems by an output regulation approach," *International Journal of Systems Science*, vol. 48, no. 10, pp. 2041–2054, 2017.

[7] W. Zou and Z. Xiang, "Event-triggered containment control of second-order nonlinear multi-agent systems," *Journal of the Franklin Institute*, 2018.

[8] B. L. Zhang, Q. L. Han, and X. M. Zhang, "Event-triggered H_∞ reliable control for offshore structures in network environments," *Journal of Sound and Vibration*, vol. 368, pp. 1–21, 2016.

[9] H. Wang, A. Xue, J. Wang, and R. Lu, "Event-based H_∞ filtering for discrete-time markov jump systems with network-induced delay," *Journal of the Franklin Institute*, vol. 354, no. 14, p. S0016003217303319, 2017.

[10] W. Huijiao and X. Anke, "Adaptive event-triggered H_∞ filtering for discrete-time delayed neural networks with randomly occurring missing measurements," *Signal Processing*, vol. 153, pp. S0 165 168 418 302 597–, 2018.

[11] D. Du, Q. Bo, M. Fei, and Z. Wang, "Quantized control of distributed event-triggered networked control systems with hybrid wired-wireless networks communication constraints," *Information Sciences An International Journal*, vol. 380, no. C, pp. 74–91, 2017.

[12] Z. Gu, P. Shi, D. Yue, and Z. Ding, "Decentralized adaptive event-triggered H_∞ filtering for a class of networked nonlinear interconnected systems," *IEEE Transactions on Cybernetics*, vol. 49, no. 5, pp. 1570–1579, 2019.

[13] E. Tian, Z. Wang, L. Zou, and D. Yue, "Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 5, pp. 1484–1498, 2019.

[14] W. Zou, C. K. Ahn, and Z. Xiang, "Event-triggered consensus tracking control of stochastic nonlinear multiagent systems," *IEEE Systems Journal*, pp. 1–9, 2019.

[15] T. C. Lee and Z. P. Jiang, "Uniform asymptotic stability of nonlinear switched systems with an application to mobile robots," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1235–1252, 2008.

[16] X. Zhao, L. Zhang, P. Shi, and H. R. Karimi, "Robust control of continuous-time systems with state-dependent uncertainties and its application to electronic circuits," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 8, pp. 4161–4170, 2014.

[17] B. Lu, F. Wu, and S. Kim, "Switching LPV control of an F-16 aircraft via controller state reset," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 2, pp. 267–277, 2006.

[18] W. Zou, P. Shi, Z. Xiang, and Y. Shi, "Consensus tracking control of switched stochastic nonlinear multiagent systems via event-triggered strategy," *IEEE Transactions on Neural Networks and Learning Systems*, pp. 1–10, 2019.

[19] W. Zou and Z. Xiang, "Event-triggered leaderfollowing consensus of non-linear multi-agent systems with switched dynamics," *IET Control Theory Applications*, vol. 13, no. 9, pp. 1222–1228, 2019.

[20] G. Ma, X. Liu, L. Qin, and G. Wu, "Finite-time event-triggered H_∞ control for switched systems with time-varying delay," *Neurocomputing*, vol. 207, pp. 828 – 842, 2016.

[21] S. Wang, M. Zeng, J. H. Park, L. Zhang, T. Hayat, and A. Alsaedi, "Finite-time control for networked switched linear systems with an event-driven communication approach," *International Journal of Systems Science*, vol. 48, no. 2, pp. 236–246, 2017.

[22] Y. Qi and M. Cao, "Finite-time boundedness and stabilisation of switched linear systems using event-triggered controllers," *IET Control Theory Applications*, vol. 11, no. 18, pp. 3240–3248, 2017.

[23] H. Ren, G. Zong, and T. Li, "Event-triggered finite-time control for networked switched linear systems with asynchronous switching," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 11, pp. 1874–1884, 2018.

[24] T. Li, J. Fu, F. Deng, and T. Chai, "Stabilization of switched linear neutral systems: An event-triggered sampling control scheme," *IEEE Transactions on Automatic Control*, vol. 63, no. 10, pp. 3537–3544, 2018.

[25] X. Xiao, L. Zhou, D. W. C. Ho, and G. Lu, "Event-triggered control of continuous-time switched linear systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1710–1717, 2019.

[26] L. Feng and Y. Song, "Stability condition for sampled data based control of linear continuous switched systems," *Systems and Control Letters*, vol. 60, no. 10, pp. 787 – 797, 2011.

[27] L. Weiss and E. Infante, "Finite time stability under perturbing forces and on product spaces," *IEEE Transactions on Automatic Control*, vol. 12, no. 1, pp. 54–59, 1967.

[28] J. Cheng, H. Zhu, S. Zhong, F. Zheng, and Y. Zeng, "Finite-time filtering for switched linear systems with a mode-dependent average dwell time," *Nonlinear Analysis: Hybrid Systems*, vol. 15, pp. 145 – 156, 2015.

[29] Y. Dong, W. Liu, T. Li, and S. Liang, "Finite-time boundedness analysis and H_∞ control for switched neutral systems with mixed time-varying delays," *Journal of the Franklin Institute*, vol. 354, no. 2, pp. 787 – 811, 2017.

[30] S. Shi, Z. Shi, Z. Fei, and Z. Liu, "Finite-time output feedback control for discrete-time switched linear systems with mode-dependent persistent dwell-time," *Journal of the Franklin Institute*, vol. 355, no. 13, pp. 5560 – 5575, 2018.

[31] B. Ning and H. Qing-Long, "Prescribed finite-time consensus tracking for multi-agent systems with nonholonomic chained-form dynamics," *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1–1, 2018.

[32] S. Huang, Z. Yan, G. Zeng, Z. Zhang, and Z. Zhu, "Finite-time stabilization of a class of upper-triangular switched nonlinear systems," *Journal of the Franklin Institute*, vol. 356, no. 6, pp. 3398 – 3418, 2019.

[33] Z. Wang, D. Wang, B. Shen, and F. E. Alsaadi, "Centralized security-guaranteed filtering in multirate-sensor fusion under deception attacks," *Journal of the Franklin Institute*, vol. 355, no. 1, pp. 406 – 420, 2018.

[34] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Observer-based event-triggering consensus control for multiagent systems with lossy sensors and cyber-attacks," *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 1936–1947, 2017.

[35] D. Ding, Q.-L. Han, Y. Xiang, X. Ge, and X.-M. Zhang, "A survey on security control and attack detection for industrial cyber-physical systems," *Neurocomputing*, vol. 275, pp. 1674 – 1683, 2018.

[36] J. Liu, Y. Gu, L. Zha, Y. Liu, and J. Cao, "Event-triggered H_∞ load frequency control for multiarea power systems under hybrid cyber attacks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. PP, no. 99, pp. 1–14, 2019.

[37] Z. Gu, X. Zhou, T. Zhang, F. Yang, and M. Shen, "Event-triggered filter design for nonlinear cyberphysical systems subject to deception attacks," *ISA Transactions*, 2019.

[38] D. Du, X. Li, W. Li, R. Chen, M. Fei, and L. Wu, "Admm-based distributed state estimation of smart grid under data deception and denial of service attacks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pp. 1–14, 2019.

[39] J. Liu, W. Lili, X. Xiangpeng, T. Engang, and F. Shumin, "Quantized stabilization for t-s fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3820–3834, 2018.

[40] E. Tian and D. Yue, "Decentralized control of network-based interconnected systems: A state-dependent triggering method," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 8, pp. 1126–1144, 2015.

[41] C. K. Zhang, Y. He, L. Jiang, and M. Wu, "Stability analysis for delayed neural networks considering both conservativeness and complexity," *IEEE Transactions on Neural Networks Learning Systems*, vol. 27, no. 7, pp. 1486–1501, 2017.

[42] Y. Dong, T. Engang, Z. Yijun, and P. Chen, "Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay," *IEEE Transactions on Systems Man and Cybernetics Part B Cybernetics A Publication of the IEEE Systems Man and Cybernetics Society*, vol. 39, no. 2, pp. 503–16, 2009.